SOLUTION OF EXERCISE # 1.3

Exercise # 1.3

Q.1: Without solving, Find the sum and product of the roots of the following equations.

(i) $2x^2 + 5x - 1 = 0$

Sol. $2x^2 + 5x - 1 = 0$

Here $a = 2,\quad b = 5,\quad c = -1$

Sum of roots

$$S = -\frac{b}{a} = \boxed{-\frac{5}{2}}$$

Product of roots

$$P = \frac{c}{a} = \boxed{-\frac{1}{2}}$$

(ii) $x^2 - 9 = 0$ **(IA-2017), (IIA-2017)**

Sol. $x^2 - 9 = 0$

Here $a = 1,\quad b = 0,\quad c = -9$

Sum of roots

$$S = -\frac{b}{a} = -\frac{0}{1} = \boxed{0}$$

Product of roots

$$P = \frac{c}{a} = \frac{-9}{1} = \boxed{-9}$$

(iii) $2x^2 + 4 = 7x$ **(IA-2022)**

Sol. $2x^2 + 4 = 7x$

$2x^2 - 7x + 4 = 0$

Here $a = 2,\quad b = -7,\quad c = 4$

Sum of roots

$$S = -\frac{b}{a} = -\frac{-7}{2} = \boxed{\frac{7}{2}}$$

Product of roots

$$P = \frac{c}{a} = \frac{4}{2} = \boxed{2}$$

(iv) $5x^2 + x - 7 = 0$ **(IIA-2019)**

Sol. Here $a = 5,\quad b = 1,\quad c = -7$

Sum of roots

$$S = -\frac{b}{a} = \boxed{-\frac{1}{5}}$$

Product of roots

$$P = \frac{c}{a} = \boxed{-\frac{7}{5}}$$

Q.2: Find the value of k, given that:

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(i) The product of the roots of the equation,

$$(k+1)x^2+(4k+3)x+(k-1)=0 \text{ is } =\frac{7}{2}$$ **(IIA-2020)**

Sol. $(k+1)x^2+(4k+3)x+(k-1)=0$

Here $a=(k+1)$, $b=(4k+3)$, $c=(k-1)$

As, Product of roots $=\frac{7}{2}$

$$\frac{c}{a}=\frac{7}{2}$$

$$\frac{k-1}{k+1}=\frac{7}{2}$$

$$2(k-1)=7(k+1)$$

$$2k-2=7k+7$$

$$2k-7k=7+2$$

$$-5k=9 \quad\Rightarrow\quad \boxed{k=-\frac{9}{5}}$$

(ii) The sum of the roots of the equation $3x^2+kx+5=0$ will be equal to the product of its roots. **(IA-2017), (IIA-2017), (IIA-2018)**

Sol. $3x^2+kx+5=0$

Here $a=3$, $b=k$, $c=5$

As, Sum of roots = product of roots

$$\frac{-b}{a}=\frac{c}{a}$$

$$\frac{-k}{3}=\frac{5}{3}$$

$$-k=5 \quad\Rightarrow\quad \boxed{k=-5}$$

(iii) The sum of the roots of the equation $4x^2+kx-7=0$ is 3. **(IA-2018), (IIA-2020)**

Sol. $4x^2+kx-7=0$

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Here $a = 4$, $b = k$, $c = -7$

As, Sum of roots = 3

$$-\frac{b}{a} = 3$$

$$-\frac{k}{4} = 3 \Rightarrow -k = 12 \Rightarrow \boxed{k = -12}$$

Q.3: (i) If the difference of the roots $x^2 - 7x + k - 4 = 0$ is 5, find the values of k and the roots. (IIA-2016), (IA-2018)

Sol. Let α *and* β are roots of the given equation,

$x^2 - 7x + k - 4 = 0$

Here $a = 1$, $b = -7$, $c = k - 4$

According to condition

$\alpha - \beta = 5$ ______ (i)

Sum of roots $= \frac{-b}{a}$

$$\alpha + \beta = -\frac{-7}{1} = 7 \rightarrow \text{(ii)}$$

Product of roots $= \frac{c}{a}$

$$\alpha\beta = \frac{k-4}{1} = k - 4 \rightarrow \text{(iii)}$$

Adding eq. (i) & eq. (ii)

$$\begin{aligned} \alpha - \beta &= 5 \\ \alpha + \beta &= 7 \\ \hline 2\alpha &= 12 \end{aligned}$$

$$\alpha = \frac{12}{2} \Rightarrow \boxed{\alpha = 6}$$

Put $\alpha = 6$ in eq.(i)

$6 - \beta = 5$

$-\beta = 5 - 6$

$-\beta = -1$

$\boxed{\beta = 1}$

so, eq.(iii) becomes

$\alpha\beta = k - 4$

$6(1) = k - 4$

$6 = k - 4 \Rightarrow k = 6 + 4$

$\boxed{k = 10}$

(ii) If the difference of the roots, $6x^2 - 23x + k = 0$ is $\frac{5}{6}$, find the values of k and the roots.

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Sol. Let α *and* β are roots of the given equation

$6x^2 - 23x + k = 0$

Here $a = 6$, $b = -23$, $c = k$

According to the condition,

$$\alpha - \beta = \frac{5}{6} \quad \text{\_\_\_\_\_\_} \ (i)$$

Sum of roots $= \frac{-b}{a}$

$$\alpha + \beta = -\frac{-23}{6} = \frac{23}{6} \rightarrow (ii)$$

Product of roots $= \frac{c}{a}$

$$\alpha\beta = \frac{k}{6} \rightarrow (iii)$$

Adding eq.(i) & eq.(ii)

$$\alpha - \beta = \frac{5}{6}$$

$$\alpha + \beta = \frac{23}{6}$$

$$2\alpha = \frac{5}{6} + \frac{23}{6}$$

$$2\alpha = \frac{5+23}{6}$$

$$\alpha = \frac{28}{12} \quad \Rightarrow \quad \boxed{\alpha = \frac{7}{3}}$$

Put $\alpha = \frac{7}{2}$ in eq.(i)

$$\frac{7}{3} - \beta = \frac{5}{6}$$

$$-\beta = \frac{5}{6} - \frac{7}{3}$$

$$-\beta = \frac{5-14}{6} = \frac{-9}{6}$$

$$-\beta = \frac{-3}{2} \quad \Rightarrow \quad \boxed{\beta = 3/2}$$

so, eq.(iii) becomes

$$\alpha\beta = \frac{k}{6}$$

$$\left(\frac{7}{3}\right)\left(\frac{3}{2}\right) = \frac{k}{6}$$

$$\frac{7}{2} = \frac{k}{6}$$

$$\frac{7}{2} \times 6 = k$$

$$7 \times 3 = k \Rightarrow \boxed{k = 21}$$

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Q.4: If α, β are the roots of $ax^2 + bx + c = 0$, find the values of:

(i) $\alpha^3 + \beta^3$

Sol. $ax^2 + bx + c = 0$

Here $a = a$, $b = b$, $c = c$

$$\text{Sum of roots} = \alpha + \beta = \frac{-b}{a}$$

$$\text{Product of roots} = \alpha\beta = \frac{c}{a}$$

$$\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$$

$$= \left(\frac{-b}{a}\right)^3 - 3\left(\frac{c}{a}\right)\left(\frac{-b}{a}\right) = \frac{-b^3}{a^3} + \frac{3bc}{a^2} = \boxed{\frac{-b^3 + 3abc}{a^3}}$$

(ii) $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$

Sol. $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$

$$= \frac{\beta^2 + \alpha^2}{\alpha^2\beta^2}$$

$$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha^2\beta^2}$$

$$= \frac{\left(\frac{-b}{a}\right)^2 - 2\left(\frac{c}{a}\right)}{\left(\frac{c}{a}\right)^2} = \frac{\frac{b^2}{a^2} - 2\frac{c}{a}}{\frac{c^2}{a^2}}$$

$$= \frac{b^2 - 2ac}{a^2} \times \frac{a^2}{c^2} = \boxed{\frac{b^2 - 2ac}{c^2}}$$

(iii) $\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}}$

Sol. $\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}}$

$$= \frac{\sqrt{\alpha}}{\sqrt{\beta}} + \frac{\sqrt{\beta}}{\sqrt{\alpha}}$$

$$= \frac{(\sqrt{\alpha})^2 + (\sqrt{\beta})^2}{\sqrt{\alpha\beta}}$$

$$= \frac{\alpha+\beta}{\sqrt{\alpha\beta}} = \frac{-\frac{b}{a}}{\sqrt{\frac{c}{a}}}$$

$$= \frac{-b}{a} \times \frac{\sqrt{a}}{\sqrt{c}} = \boxed{\frac{-b}{\sqrt{ac}}}$$

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(iv) $\dfrac{\alpha^2}{\beta}+\dfrac{\beta^2}{\alpha}$

Sol. $\dfrac{\alpha^2}{\beta}+\dfrac{\beta^2}{\alpha}$

$$=\frac{\alpha^3+\beta^3}{\alpha\beta}$$

$$=\frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{\alpha\beta}$$

$$=\frac{\left(\frac{-b}{a}\right)^3-3\left(\frac{c}{a}\right)\left(\frac{-b}{a}\right)}{\frac{c}{a}}$$

$$=\frac{-\frac{b^3}{a^3}+\frac{3bc}{a^2}}{\frac{c}{a}}$$

$$=\frac{-b^3+3abc}{a^3}\times\frac{a}{c}$$

$$=\boxed{\frac{-b^3+3abc}{a^2c}}$$

(v) $\dfrac{\alpha}{\beta}-\dfrac{\beta}{\alpha}$

Sol. $\dfrac{\alpha}{\beta}-\dfrac{\beta}{\alpha}=\dfrac{\alpha^2-\beta^2}{\alpha\beta}$

$$=\frac{(\alpha+\beta)(\alpha-\beta)}{\alpha\beta}$$

$$=\frac{(\alpha+\beta)\sqrt{(\alpha-\beta)^2}}{\alpha\beta}$$

$$=\frac{(\alpha+\beta)\sqrt{\alpha^2+\beta^2-2\alpha\beta}}{\alpha\beta}$$

$$=\frac{(\alpha+\beta)\sqrt{(\alpha+\beta)^2-4\alpha\beta}}{\alpha\beta}$$

$$=\frac{\left(\frac{-b}{a}\right)\sqrt{\left(\frac{-b}{a}\right)^2-4\left(\frac{c}{a}\right)}}{\frac{c}{a}}$$

$$=\frac{-b}{a}\sqrt{\frac{b^2}{a^2}-\frac{4c}{a}}\times\left(\frac{a}{c}\right)$$

$$=\frac{-b}{c}\sqrt{\frac{b^2-4ac}{a^2}}=\boxed{\frac{-b\sqrt{b^2-4ac}}{ac}}$$

Q.5: If p, q are the roots of $2x^2-6x+3=0$, find the value of $(p^3+q^3)-3pq(p^2+q^2)-3pq(p+q)$

Sol. As p, q are the roots of the given equations

$$2x^2-6x+3=0$$

Here $a=2,\qquad b=-6,\qquad c=3$

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Sum of roots $= -\dfrac{b}{a}$	Product of roots $= \dfrac{c}{a}$
$p+q = \dfrac{-6}{2} = 3$	$pq = \dfrac{3}{2}$

$(p^3 + q^3) - 3pq\,(p^2 + q^2) - 3pq\,(p + q)$

$= (p+q)^3 - 3pq\,(p+q) - 3pq((p+q)^2 - 2pq) - 3pq\,(p+q)$

$$= 3^3 - 3\left(\frac{3}{2}\right)3 - 3\left(\frac{3}{2}\right)\left[(3)^2 - 2\left(\frac{3}{2}\right)\right] - 3\left(\frac{3}{2}\right)3$$

$$= 27 - \frac{27}{2} - \frac{9}{2}(9-3) - \frac{27}{2}$$

$$= = 27 - \frac{27}{2} - \frac{9}{2}(6) - \frac{27}{2}$$

$$= \cancel{27} - \frac{27}{2} - \cancel{27} - \frac{27}{2}$$

$$= = -\frac{27}{2} - \frac{27}{2} = \frac{-27-27}{2} = \frac{-54}{2} = \boxed{-27}$$

Q.6: The roots of the equation $px^2 + qx + q = 0$ are α and β, prove that: $\sqrt{\dfrac{\alpha}{\beta}} + \sqrt{\dfrac{\beta}{\alpha}} + \sqrt{\dfrac{q}{p}} = 0$ (IIA-2018)

Sol. $px^2 + qx + q = 0$

Here $a = p$, $b = q$, $c = q$

Sum of roots $= -\dfrac{b}{a}$	Product of roots $= \dfrac{c}{a}$
$\Rightarrow \quad \alpha + \beta = -\dfrac{q}{p}$	$\Rightarrow \quad \alpha\beta = \dfrac{q}{p}$

$$\text{L.H.S.} = \sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{q}{p}}$$

$$= \frac{\sqrt{\alpha}}{\sqrt{\beta}} + \frac{\sqrt{\beta}}{\sqrt{\alpha}} + \sqrt{\frac{q}{p}}$$

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$$=\frac{\left(\sqrt{\alpha}\right)^2+\left(\sqrt{\beta}\right)^2}{\sqrt{\alpha}\times\sqrt{\beta}}+\sqrt{\frac{q}{p}}=\frac{\alpha+\beta}{\sqrt{\alpha\beta}}+\sqrt{\frac{q}{p}}$$

$$=\frac{\frac{-q}{p}}{\sqrt{\frac{q}{p}}}+\sqrt{\frac{q}{p}}=\frac{\frac{-q}{p}+\left(\sqrt{\frac{q}{p}}\right)^2}{\sqrt{\frac{q}{p}}}$$ by taking L.C.M

$$=\frac{-\frac{q}{p}+\frac{q}{p}}{\sqrt{\frac{q}{p}}}=\frac{0}{\sqrt{\frac{q}{p}}}=0=\text{R.H.S.}$$ **Proved**

Q.7: Find the condition that one root of the equation $px^2+qx+r=0$ is square of the other.

Sol. According to the condition, α, α^2 are the roots of given equation. $px^2+qx+r=0$

Here $a=p$, $b=q$, $c=r$

Sum of Roots	Products of Roots
$\text{Sum of Roots}=\frac{-b}{a}$	$\text{Products of Roots}=\frac{c}{a}$
$\alpha+\alpha^2=-\frac{q}{p}\rightarrow(\text{i})$	$\alpha.\alpha^2=\frac{r}{p}\Rightarrow\alpha^3=\frac{r}{p}\rightarrow(\text{ii})$

Taking Cube on both sides of eq. (i), we have

$$\left(\alpha+\alpha^2\right)^3=\left(-\frac{q}{p}\right)^3$$

$$(\alpha)^3+\left(\alpha^2\right)^3+3(\alpha)\left(\alpha^2\right)\left(\alpha+\alpha^2\right)=-\frac{q^3}{p^3}$$

$$\alpha^3+\alpha^6+3\alpha^3\left(\alpha+\alpha^2\right)=-\frac{q^3}{p^3}$$

$$\left(\frac{r}{p}\right)+\left(\frac{r}{p}\right)^2+3\frac{r}{p}\left(-\frac{q}{p}\right)=-\frac{q^3}{p^3}$$ $\because$ using eq.(i) & eq.(ii)

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$$\frac{r}{p}+\frac{r^2}{p^2}-\frac{3rq}{p^2}=-\frac{q^3}{p^3}$$

Multipling both sides by p^3, we get

$$p^2r+pr^2-3pqr=-q^3$$

$$p^2r+pr^2+q^3=3pqr \Rightarrow \boxed{pr(p+r)+q^3=3pqr}$$

Q.8: Find the value of k given that if one root of $9x^2-15x+k=0$ exceeds the other by 3. Also find the roots. (IA-2016)

Sol. According to the condition, α, $\alpha+3$ are the roots of given equation: $9x^2-15x+k=0$

Here $a=9$, $b=-15$, $c=k$

Sum of Roots $=\frac{-b}{a}$	Products of Roots $=\frac{c}{a}$
$\alpha+\alpha+3=-\left(\frac{-15}{9}\right)$	$(\alpha)(\alpha+3)=\frac{k}{9}$
$2\alpha+3=\frac{15}{9}\rightarrow(i)$	$\alpha^2+3\alpha=\frac{k}{9}\rightarrow(ii)$

From eq.(i) $2\alpha=\frac{15}{9}-3$

$$2\alpha=\frac{15-27}{9} \qquad \alpha=\frac{-12}{18}=\frac{-2}{3}$$

Put $\alpha=-\frac{2}{3}$ in eq.(ii), we have

$$\left(\frac{-2}{3}\right)^2+3\left(\frac{-2}{3}\right)=\frac{k}{9}$$

$$\frac{4}{9}-2=\frac{k}{9} \Rightarrow \frac{4-18}{9}=\frac{k}{9}$$

$$-14=k \text{ or } \boxed{k=-14}$$

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So, one roots is $= \alpha = \boxed{\frac{-2}{3}}$

and other root is $= \alpha + 3 = \frac{-2}{3} + 3 = \frac{-2+9}{3} = \boxed{\frac{7}{3}}$

Q.9: If α, β are the roots of the equation $px^2 + qx + r = 0$ then find the values of:

Sum of roots $= -\frac{b}{a}$

$\Rightarrow \alpha + \beta = -\frac{q}{p}$

Product of roots $= \frac{c}{a}$

$\Rightarrow \alpha\beta = \frac{r}{p}$

(i) $\alpha^2 + \beta^2$

Sol. $\alpha^2 + \beta^2$

$= (\alpha + \beta)^2 - 2\alpha\beta$

$= \left(-\frac{q}{p}\right)^2 - 2\left(\frac{r}{p}\right)$

$= \frac{q^2}{p^2} - \frac{2r}{p} = \boxed{\frac{q^2 - 2pr}{p^2}}$

(ii) $(\alpha - \beta)^2$

Sol. $\alpha^2 + \beta^2 - 2\alpha\beta$

$= (\alpha + \beta)^2 - 2\alpha\beta - 2\alpha\beta$

$= (\alpha + \beta)^2 - 4\alpha\beta$

$= \left(-\frac{q}{p}\right)^2 - 4\left(\frac{r}{p}\right)$

$= \frac{q^2}{p^2} - \frac{4r}{p} = \boxed{\frac{q^2 - 4pr}{p^2}}$

(iii) $\alpha^3\beta + \alpha\beta^3$

Sol. $\alpha^3\beta + \alpha\beta^3$

$= \alpha\beta(\alpha^2 + \beta^2)$

$= \alpha\beta\left((\alpha + \beta)^2 - 2\alpha\beta\right)$

$= \frac{r}{p}\left(\left(-\frac{q}{p}\right)^2 - 2\left(\frac{r}{p}\right)\right)$

$= \frac{r}{p}\left(\frac{q^2}{p^2} - \frac{2r}{p}\right)$

$= \frac{r}{p}\left(\frac{q^2 - 2pr}{p^2}\right)$

$= \boxed{\frac{r(q^2 - 2pr)}{p^3}}$